Outline

- Part 1: Motivation
- Part 2: Probabilistic Databases
- Part 3: Weighted Model Counting
- Part 4: Lifted Inference for WFOMC
- Part 5: Completeness of Lifted Inference
- Part 6: Query Compilation
- Part 7: Symmetric Lifted Inference Complexity
- Part 8: Open-World Probabilistic Databases
- Part 9: Discussion & Conclusions

What Everyone Should Know about Databases

- Database = several relations (a.k.a. tables)
- SQL Query = FO Formula
- Boolean Query = FO Sentence

What Everyone Should Know about Databases

Database: relations (= tables)

D =

Smoker

X	Y
Alice	2009
Alice	2010
Bob	2009
Carol	2010

Friend

X	Z
Alice	Bob
Alice	Carol
Bob	Carol
Carol	Bob

What Everyone Should Know about Databases

Database: relations (= tables)

D =

Smoker

x	y
Alice	2009
Alice	2010
Bob	2009
Carol	2010

Friend

x	z
Alice	Bob
Alice	Carol
Bob	Carol
Carol	Bob

Query: First Order Formula

$Q(z) = \exists x\,(\text{Smoker}(x,\text{'2009'}) \wedge \text{Friend}(x,z))$

Find friends of smokers in 2009

Query answer: Q(D) =

z
Bob
Carol

Conjunctive Queries CQ = FO(∃, ∧)

Union of CQs UCQ = FO(∃, ∧, ∨)

What Everyone Should Know about Databases

Database: relations (= tables)

D =

Smoker

X	Y
Alice	2009
Alice	2010
Bob	2009
Carol	2010

Friend

X	Z
Alice	Bob
Alice	Carol
Bob	Carol
Carol	Bob

Query: First Order Formula

$Q(z) = \exists x\,(\text{Smoker}(x,\text{'2009'}) \wedge \text{Friend}(x,z))$

Find friends of smokers in 2009

Query answer: Q(D) =

Z
Bob
Carol

Conjunctive Queries CQ = FO(∃, ∧)

Union of CQs UCQ = FO(∃, ∧, ∨)

Boolean Query: FO Sentence

$Q = \exists x\,(\text{Smoker}(x,\text{'2009'}) \wedge \text{Friend}(x,\text{'Bob'}))$

Query answer: Q(D) = TRUE

What Everyone Should Know about Databases

Declarative Query	→	Query Plan
"what"	→	*"how"*

What Everyone Should Know about Databases

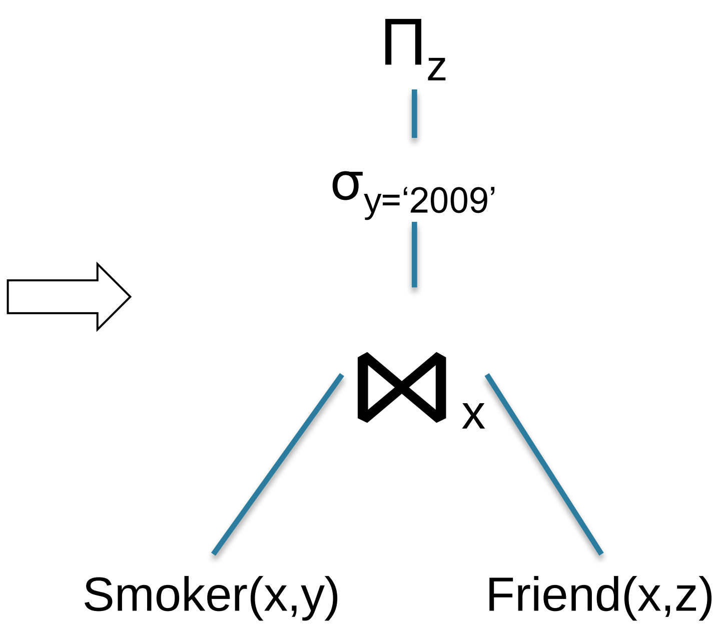
Declarative Query	→	Query Plan
"what"	→	*"how"*

$Q(z) = \exists x\, (\text{Smoker}(x,\text{'2009'}) \wedge \text{Friend}(x,z))$

What Everyone Should Know about Databases

Declarative Query	→	Query Plan
"what"	→	*"how"*

$Q(z) = \exists x\,(\text{Smoker}(x,\text{'2009'}) \wedge \text{Friend}(x,z))$

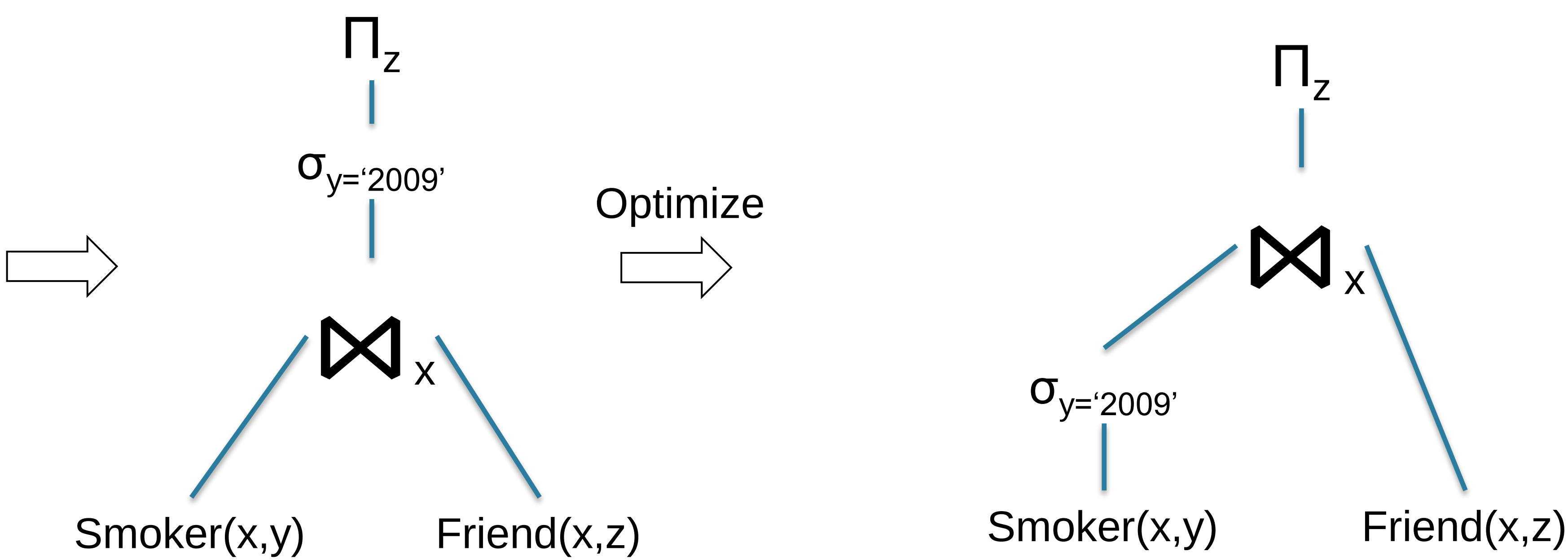


Logical Query Plan

What Everyone Should Know about Databases

Declarative Query	→	Query Plan
"what"	→	*"how"*

$Q(z) = \exists x\,(\text{Smoker}(x,\text{'2009'}) \wedge \text{Friend}(x,z))$



Logical Query Plan

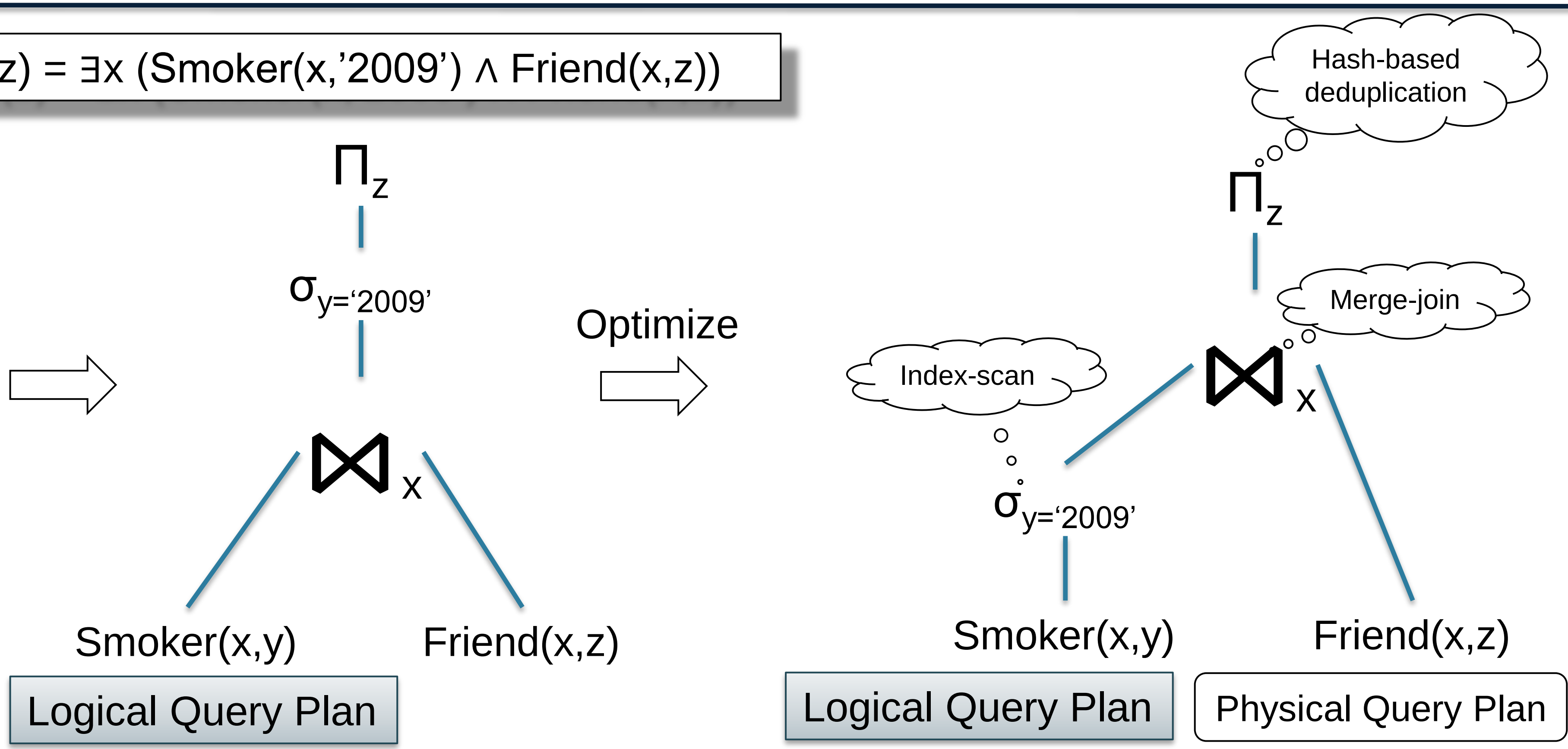
Logical Query Plan

What Everyone Should Know about Databases

Declarative Query → Query Plan

"what" → *"how"*

$Q(z) = \exists x\, (Smoker(x,'2009') \wedge Friend(x,z))$



What Every Researcher Should Know about Databases

Problem: compute Q(D)

Moshe Vardi [Vardi'82]
2008 ACM SIGMOD Contribution Award

This talk: query = blue, data = red

What Every Researcher Should Know about Databases

Problem: compute Q(D)

Moshe Vardi [Vardi'82]
2008 ACM SIGMOD Contribution Award

- Data complexity:
  fix Q, complexity = f(D)

This talk: query = blue, data = red

What Every Researcher Should Know about Databases

Problem: compute Q(D)

Moshe Vardi [Vardi'82]
2008 ACM SIGMOD Contribution Award

- Data complexity:
  fix Q, complexity = f(D)
- Query complexity: (expression complexity)
  fix D, complexity = f(Q)
- Combined complexity:
  complexity = f(D, Q)

This talk: query = blue, data = red

Probabilistic Databases

- A probabilistic database = relational database where each tuple is a random variable

- Semantics = probability distribution over possible worlds (deterministic databases)

- In this talk: tuples are independent events

Example

Probabilistic database D:

Friend

x	y	P
A	B	p_1
A	C	p_2
B	C	p_3

Example

Probabilistic database D:

Friend

x	y	P
A	B	p_1
A	C	p_2
B	C	p_3

Possible worlds semantics:

x	y
A	B
A	C
B	C

$p_1 p_2 p_3$

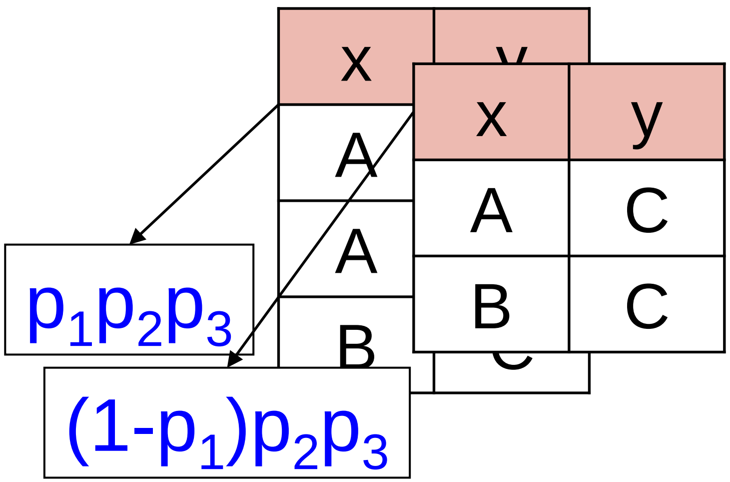
Example

Probabilistic database D:

Friend

x	y	P
A	B	p_1
A	C	p_2
B	C	p_3

Possible worlds semantics:



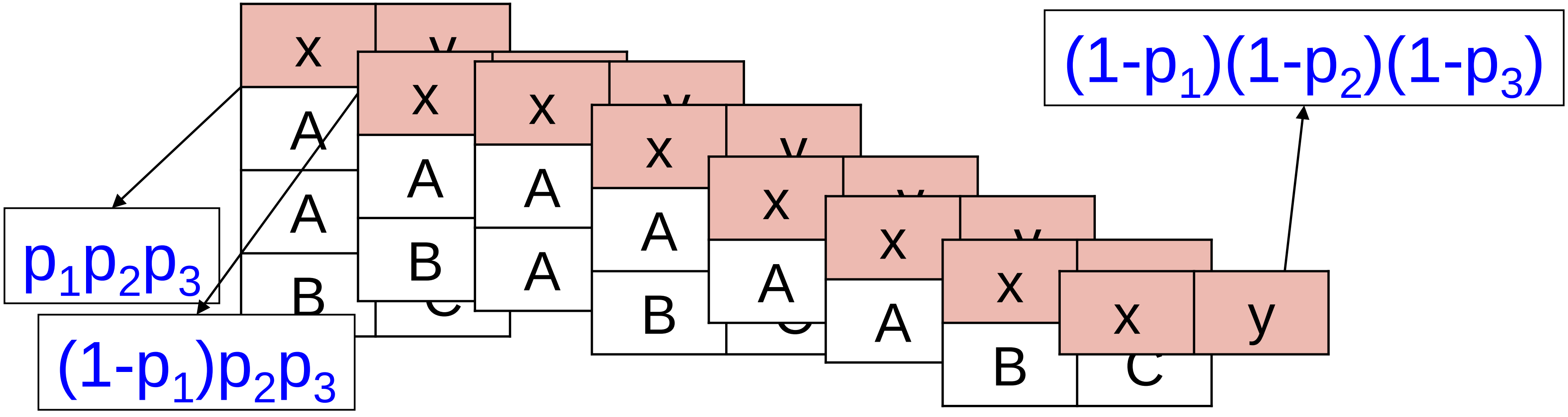
Example

Probabilistic database D:

Friend

x	y	P
A	B	p_1
A	C	p_2
B	C	p_3

Possible worlds semantics:



Query Semantics

Fix a Boolean query Q, probabilistic database D:

$P(Q \mid D) = P_D(Q)$ = marginal probability of Q on possible words of D

$Q = \exists x \exists y \; Smoker(x) \wedge Friend(x,y)$

An Example

$P(Q \mid D) =$

Friend

x	y	P
A	D	q_1
A	E	q_2
B	F	q_3
B	G	q_4
B	H	q_5

Smoker

x	P
A	p_1
B	p_2
C	p_3

$Q = \exists x \exists y \; Smoker(x) \wedge Friend(x,y)$

An Example

$P(Q \mid D) = \quad 1-(1-q_1)*(1-q_2)$

Friend

x	y	P
A	D	q_1
A	E	q_2
B	F	q_3
B	G	q_4
B	H	q_5

Smoker

x	P
A	p_1
B	p_2
C	p_3

$Q = \exists x \exists y \; Smoker(x) \wedge Friend(x,y)$

An Example

$P(Q \mid D) = p_1 * [\;\; 1-(1-q_1)*(1-q_2) \;\;]$

Smoker

x	P
A	p_1
B	p_2
C	p_3

Friend

x	y	P
A	D	q_1
A	E	q_2
B	F	q_3
B	G	q_4
B	H	q_5

$Q = \exists x \exists y\ \text{Smoker}(x) \land \text{Friend}(x,y)$

An Example

$$P(Q \mid D) = p_1 * [\ 1-(1-q_1)*(1-q_2)\ ]$$
$$1-(1-q_3)*(1-q_4)*(1-q_5)$$

Friend

x	y	P
A	D	q_1
A	E	q_2
B	F	q_3
B	G	q_4
B	H	q_5

Smoker

x	P
A	p_1
B	p_2
C	p_3

$Q = \exists x \exists y\ Smoker(x) \wedge Friend(x,y)$

An Example

$$P(Q \mid D) = p_1 * [\ 1-(1-q_1)*(1-q_2)\ ]$$
$$p_2 * [\ 1-(1-q_3)*(1-q_4)*(1-q_5)\ ]$$

Smoker

x	P
A	p_1
B	p_2
C	p_3

Friend

x	y	P
A	D	q_1
A	E	q_2
B	F	q_3
B	G	q_4
B	H	q_5

$Q = \exists x \exists y \; Smoker(x) \wedge Friend(x,y)$

An Example

$$P(Q \mid D) = 1 - \{1 - p_1 * [\; 1-(1-q_1)*(1-q_2) \;]\} * \{1 - p_2 * [\; 1-(1-q_3)*(1-q_4)*(1-q_5) \;]\}$$

Smoker

x	P
A	p_1
B	p_2
C	p_3

Friend

x	y	P
A	D	q_1
A	E	q_2
B	F	q_3
B	G	q_4
B	H	q_5

$Q = \exists x \exists y \; \text{Smoker}(x) \land \text{Friend}(x,y)$

An Example

$$P(Q \mid D) = 1 - \{1 - p_1 * [\, 1 - (1-q_1)*(1-q_2) \,]\} * \{1 - p_2 * [\, 1 - (1-q_3)*(1-q_4)*(1-q_5) \,]\}$$

One can compute P(Q | D) in PTIME in the size of the database D

Friend

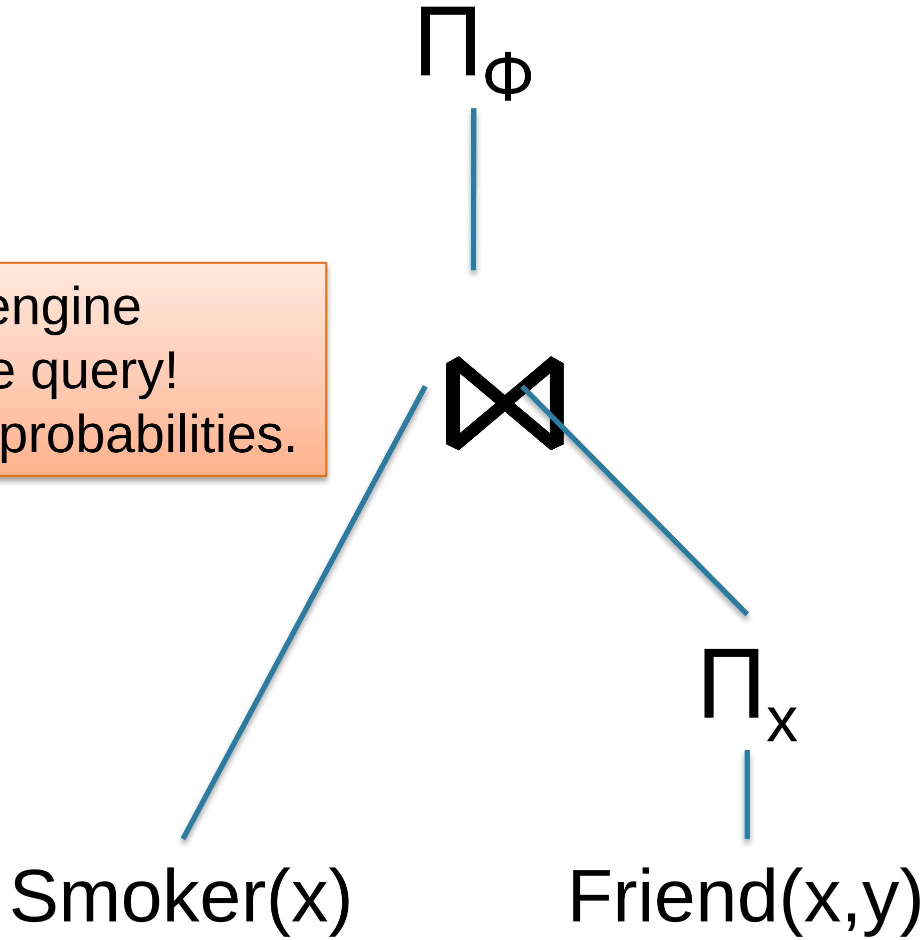
x	y	P
A	D	q_1
A	E	q_2
B	F	q_3
B	G	q_4
B	H	q_5

Smoker

x	P
A	p_1
B	p_2
C	p_3

$Q = \exists x \exists y \; Smoker(x) \wedge Friend(x,y)$

An Example



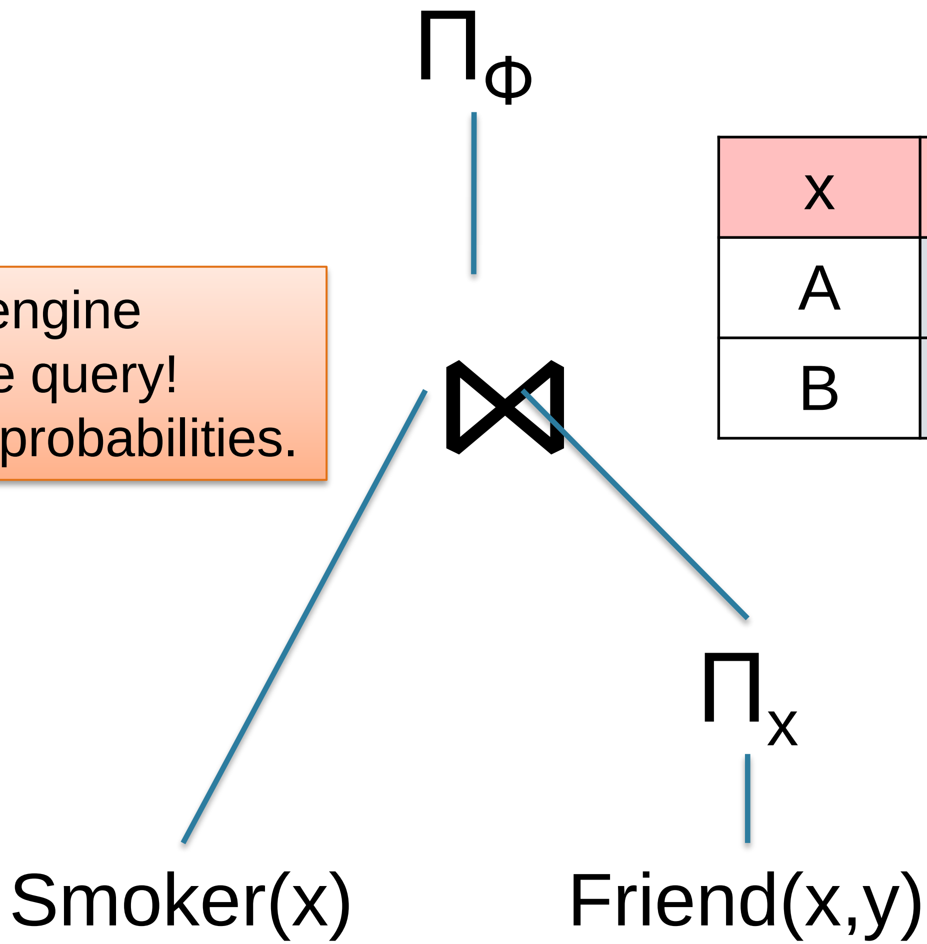
Use the SQL engine
to compute the query!
Aggregate on probabilities.

x	P
A	p_1
B	p_2
C	p_3

x	y	P
A	D	q_1
A	E	q_2
B	F	q_3
B	G	q_4
B	H	q_5

$Q = \exists x \exists y\ Smoker(x) \wedge Friend(x,y)$

An Example



Use the SQL engine
to compute the query!
Aggregate on probabilities.

x	P
A	$1-(1-q_1)(1-q_2)$
B	$1-(1-q_4)(1-q_5)\ (1-q_6)$

x	P
A	p_1
B	p_2
C	p_3

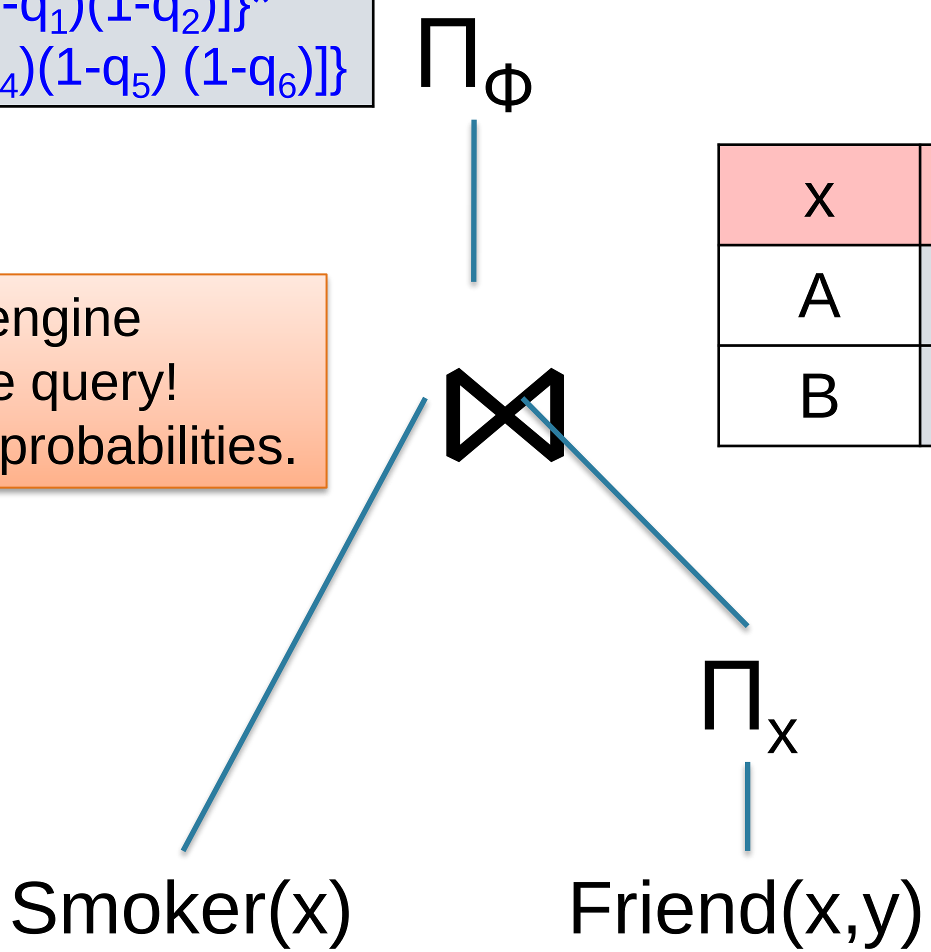
x	y	P
A	D	q_1
A	E	q_2
B	F	q_3
B	G	q_4
B	H	q_5

$Q = \exists x \exists y \; Smoker(x) \wedge Friend(x,y)$

An Example

$1-\{1-p_1[1-(1-q_1)(1-q_2)]\}*$
$\{1-p_2[1-(1-q_4)(1-q_5)\,(1-q_6)]\}$

Use the SQL engine
to compute the query!
Aggregate on probabilities.



x	P
A	$1-(1-q_1)(1-q_2)$
B	$1-(1-q_4)(1-q_5)\,(1-q_6)$

x	P
A	p_1
B	p_2
C	p_3

x	y	P
A	D	q_1
A	E	q_2
B	F	q_3
B	G	q_4
B	H	q_5

Problem Statement

Given: probabilistic database D, query Q

Compute: P(Q | D)

Data complexity: fix Q, complexity = f(|D|)

Approaches to Compute P(Q | D)

- Propositional inference:
  - Ground the query Q → $F_{Q,D}$, compute $P(F_{Q,D})$
  - This is Weighted Model Counting (later…)
  - Works for every query Q
  - But: may be exponential in |D| (data complexity)
- Lifted inference:
  - Compute a query plan for Q, execute plan on D
  - Always polynomial time in |D| (data complexity)
  - But: does not work for all queries Q

[Olteanu'08, Jha'13, Dalvi'04, Dalvi'12]

Lifted Inference Rules

Preprocess Q (omitted from this talk; see [Suciu'11]), then apply these rules (some have preconditions)

$$P(\neg Q) = 1 - P(Q)$$

negation

$$P(Q1 \wedge Q2) = P(Q1)P(Q2)$$
$$P(Q1 \vee Q2) = 1 - (1 - P(Q1))(1 - P(Q2))$$

Independent join / union

$$P(\exists z\, Q) = 1 - \Pi_{A \in Domain} (1 - P(Q[A/z]))$$
$$P(\forall z\, Q) = \Pi_{A \in Domain} P(Q[A/z])$$

Independent project

$$P(Q1 \wedge Q2) = P(Q1) + P(Q2) - P(Q1 \vee Q2)$$
$$P(Q1 \vee Q2) = P(Q1) + P(Q2) - P(Q1 \wedge Q2)$$

Inclusion/ exclusion

Example

$Q = \forall x \forall y\, (\text{Smoker}(x) \lor \text{Friend}(x,y))$

$= \forall x\, (\text{Smoker}(x) \lor \forall y\, \text{Friend}(x,y))$

∀-Rule

$P(Q) = \prod_{A \in \text{Domain}} P(\text{Smoker}(A) \lor \forall y\, \text{Friend}(A,y))$

Check independence:

$\text{Smoker}(\text{Alice}) \lor \forall y\, \text{Friend}(\text{Alice},y)$

$\text{Smoker}(\text{Bob}) \lor \forall y\, \text{Friend}(\text{Bob},y)$

Example

$Q = \forall x \forall y\,(\text{Smoker}(x) \vee \text{Friend}(x,y))$

$= \forall x\,(\text{Smoker}(x) \vee \forall y\,\text{Friend}(x,y))$

∀-Rule

$P(Q) = \prod_{A \in \text{Domain}} P(\text{Smoker}(A) \vee \forall y\,\text{Friend}(A,y))$

Check independence:

Smoker(Alice) ⋁ ∀y Friend(Alice,y)

Smoker(Bob) ⋁ ∀y Friend(Bob,y)

$P(Q) = \prod_{A \in \text{Domain}} [1 - (1 - P(\text{Smoker}(A))) \times (1 - P(\forall y\,\text{Friend}(A,y)))]$

⋁-Rule

Example

$Q = \forall x \forall y\, (Smoker(x) \lor Friend(x,y))$ $= \forall x\, (Smoker(x) \lor \forall y\, Friend(x,y))$

∀-Rule

$P(Q) = \prod_{A \in Domain} P(Smoker(A) \lor \forall y\, Friend(A,y))$

Check independence:

$Smoker(Alice) \lor \forall y\, Friend(Alice,y)$

$Smoker(Bob) \lor \forall y\, Friend(Bob,y)$

$P(Q) = \prod_{A \in Domain} [1 - (1 - P(Smoker(A))) \times (1 - P(\forall y\, Friend(A,y)))]$

∨-Rule

$P(Q) = \prod_{A \in Domain} [1 - (1 - P(Smoker(A))) \times (1 - \prod_{B \in Domain} P(Friend(A,B)))]$

∀-Rule

Example

$Q = \forall x \forall y\,(\text{Smoker}(x) \vee \text{Friend}(x,y))$ $= \forall x\,(\text{Smoker}(x) \vee \forall y\,\text{Friend}(x,y))$

∀-Rule

$P(Q) = \prod_{A \in \text{Domain}} P(\text{Smoker}(A) \vee \forall y\,\text{Friend}(A,y))$

Check independence:
Smoker(Alice) ⋁ ∀y Friend(Alice,y)
Smoker(Bob) ⋁ ∀y Friend(Bob,y)

$P(Q) = \prod_{A \in \text{Domain}} [1 - (1 - P(\text{Smoker}(A))) \times (1 - P(\forall y\,\text{Friend}(A,y)))]$

⋁-Rule

$P(Q) = \prod_{A \in \text{Domain}} [1 - (1 - P(\text{Smoker}(A))) \times (1 - \prod_{B \in \text{Domain}} P(\text{Friend}(A,B)))]$

Lookup the probabilities in the database

∀-Rule

Example

$Q = \forall x \forall y\,(\text{Smoker}(x) \lor \text{Friend}(x,y))$ $= \forall x\,(\text{Smoker}(x) \lor \forall y\,\text{Friend}(x,y))$

∀-Rule

$P(Q) = \prod_{A \in \text{Domain}} P(\text{Smoker}(A) \lor \forall y\,\text{Friend}(A,y))$

Check independence:
$\text{Smoker}(\text{Alice}) \lor \forall y\,\text{Friend}(\text{Alice},y)$
$\text{Smoker}(\text{Bob}) \lor \forall y\,\text{Friend}(\text{Bob},y)$

$P(Q) = \prod_{A \in \text{Domain}} [1 - (1 - P(\text{Smoker}(A))) \times (1 - P(\forall y\,\text{Friend}(A,y)))]$

∨-Rule

$P(Q) = \prod_{A \in \text{Domain}} [1 - (1 - P(\text{Smoker}(A))) \times (1 - \prod_{B \in \text{Domain}} P(\text{Friend}(A,B)))]$

Lookup the probabilities in the database

∀-Rule

Runtime = $O(n^2)$.

Discussion: CNF vs. DNF

Databases		KR/AI	
Conjunctive Queries CQ	FO(∃, ∧)	Positive Clause	FO(∀, ∨)
Union of Conjunctive Queries UCQ	FO(∃, ∧, ∨) = ∃ Positive-DNF	Positive FO	FO(∀, ∧, ∨) = ∀ Positive-CNF
UCQ with "safe negation" UCQ¬	∃ DNF	First Order CNF	∀ CNF
Q = ∃x,∃y, Smoker(x) ∧ Friend(x,y)		Q = ∀x∀y (Smoker(x) ∨ Friend(x,y))	

$\exists x, \exists y,\ \text{Smoker}(x) \wedge \text{Friend}(x,y) \ = \ \neg\, \forall x, \forall y,\ (\neg\text{Smoker}(x) \vee \neg\text{Friend}(x,y))$

Discussion

Lifted Inference Sometimes Fails.

$H_0 = \forall x \forall y\, (\text{Smoker}(x) \vee \text{Friend}(x,y) \vee \text{Jogger}(y))$

The ∀-rule does not apply: H_0[Alice/x] and H_0[Bob/x] are dependent:

H_0[Alice/x] = $\forall y\, (\text{Smoker}(\text{Alice}) \vee \text{Friend}(\text{Alice},y) \vee \text{Jogger}(y))$

H_0[Bob/x] = $\forall y\, (\text{Smoker}(\text{Bob}) \vee \text{Friend}(\text{Bob},y) \vee \text{Jogger}(y))$

Dependent

Computing $P(H_0 \mid D)$ is #P-hard in $|D|$
(Proof: later…)

Discussion

Lifted Inference Sometimes Fails.

$H_0 = \forall x \forall y\, (\mathrm{Smoker}(x) \vee \mathrm{Friend}(x,y) \vee \mathrm{Jogger}(y))$

The ∀-rule does not apply: H_0[Alice/x] and H_0[Bob/x] are dependent:

$H_0[\mathrm{Alice}/x] = \forall y\, (\mathrm{Smoker}(\mathrm{Alice}) \vee \mathrm{Friend}(\mathrm{Alice},y) \vee \mathrm{Jogger}(y))$

$H_0[\mathrm{Bob}/x] = \forall y\, (\mathrm{Smoker}(\mathrm{Bob}) \vee \mathrm{Friend}(\mathrm{Bob},y) \vee \mathrm{Jogger}(y))$

Dependent

Computing $P(H_0 \mid D)$ is #P-hard in $|D|$
(Proof: later…)

Consequence: assuming PTIME ≠ #P, H_0 is not liftable!

Summary

- Database D = relations
- Query Q = FO
- Query plans, query optimization
- Data complexity: fix Q, complexity f(D)
- Probabilistic DB's = independent tuples
- Lifted inference: simple, but fails sometimes